ANALYSIS OF THE INFLUENCE EXERTED BY PHYSICOGEOMETRIC PARAMETERS ON THE TEMPERATURE FIELD OF AN OBJECT

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A method is proposed for studying the effect of various parameters in complex objects on the thermal regime within these objects. A study was also conducted into the error of this method.

In [1, 2] we find a study of the procedures involved in the design of thermostable optical systems. It is demonstrated that such design operations involve three stages: selection of the basic optical system, the development of structural components, and the calculation also of temperature fields and thermooptical aberrations. The relationship between the output characteristics of an instrument, relative to slight deviations in the initial parameters [3], is ordinarily studied in each of these stages.*

In the initial stage, such an analysis makes it possible to justify such an optical system, capable of operation even when some of the parameters of the system, during operation, fail to meet specifications as a consequence of fabrication imprecision or as a result of various occurrences (thermal, mechanical, etc.).

In the second stage, the analysis of the influence exerted by structural parameters on the temperature field allows us to isolate those which govern the thermal regime of the instrument. During the design stage, any alteration of these parameters makes it possible to affect the thermal regime in any required direction and to optimize the design.

In the third stage, we evaluate the reliability of the results obtained in the calculation of the temperature fields, we determine the need to refine any of the original information, we take a look at the stability of the thermal regime in the instrument being designed, and we choose the regimes with which to carry out the thermal tests, etc.

In order to solve these problems, we usually find an analytical relationship between the temperature of any region and the parameters of the object. Then, by means of differentiation over any given parameter, we determine the influence of these parameters on the thermal regime [4-8]. It is not always possible to apply such an approach, because of the absence of analytical relationships or because of limitations in the area of their application.

In the present study we propose a numerical-analytical method that is based on the concept of an influence coefficient, and this method is intended for complex objects. The temperature fields of objects are calculated in several stages, with various levels of detail, involving the utilization of a step-by-step simulation method [9].

In the first stage we determine the average-surface or average-volumetric temperature regions. The mathematical formulation of the problem involves a system of algebraic (for the steady-state thermal regime):

$$t_i \sum_{j}^{N} \sigma_{ij} - \sum_{j}^{N} t_j \sigma_{ij} = P_i, \; i = 1, \ldots, N, \tag{1}$$

or ordinary differential (for the nonsteady regime) equations [10]:

*In optics and in electrical engineering, analysis of influence is also referred to as sensitivity analysis.

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$$C_i\,dt_i/d\tau + t_i \sum_j^N \sigma_{ij} - \sum_j^N t_j \sigma_{ij} = P_i,\ i = 1,\ \ldots,\ N. \qquad (2)$$

Equations (1) and (2) have been compiled on the basis of the law of conservation of energy and taken into consideration the mutual thermal influence of N bodies in the system with energy sources P_i, temperature sources t_i and t_j, and conductivity sources σ_{ij} between the bodies i and j.

In the subsequent stages, using information about the average temperatures of the bodies, we find the value of the average surface temperature or the flow of heat from the surface of a given body. Having this information at hand, we are able to determine the temperature distribution in various regions. The mathematical formulation of this problem is in the form of a partial differential equation [11].

Thus, the problem of studying the influence exerted by parameters on the temperature field of a complex object breaks down into two parts:

analysis of the means temperature;

analysis of the temperature distribution in the regions.

Let us examine the first problem. Let there be a system of N bodies. The average temperatures t_i (i = 1, 2, ..., N) of its regions are functions of the m-parameters x_j, i.e.

$$t_i = t_i(x_1,\ \ldots,\ x_m),\ i = 1,\ \ldots,\ N. \qquad (3)$$

Applying a Taylor series to (3) and retaining here only the first terms, we obtain:

$$t_i = t_i^0 + \Delta t_i = t_i^0 + \sum_j^m \frac{\partial t_i}{\partial x_j}(x_1,\ \ldots,\ x_m)\,\Delta x_j. \qquad (4)$$

The coefficients with Δx_j characterize the level of influence exerted by the parameter x_j on the temperature t_i. We will refer to these as coefficients of influence and we denote

$$k_{ij} = \partial t_i(x_1,\ \ldots,\ x_m)/\partial x_j. \qquad (5)$$

With consideration of (5) we rewrite (4) to the form

$$t_i = t_i^0 + \Delta t_i = t_i^0 + \sum_j^m k_{ij}\,\Delta x_j. \qquad (6)$$

The change in the temperature of the regions, caused by the deviation of the parameters Δx_j (j = 1, ..., m), will be characterized as a discrepancy function $\varepsilon = \{\varepsilon_1,\ \ldots,\ \varepsilon_N\}$[12]. The components of the discrepancy vector ε_i are determined from system of equations (1) and (2).

For the steady-state regime

$$\varepsilon_i = P_i - \sum_j^N \sigma_{ij}(t_i - t_j),\ i = 1,\ \ldots\ N, \qquad (7)$$

and for the nonsteady regime

$$\varepsilon_i = P_i - C_i\,dt_i/d\tau - \sum_j^N \sigma_{ij}(t_i - t_j),\ i = 1,\ \ldots,\ N. \qquad (8)$$

Let us apply a Taylor series expansion to the ε_i components of the discrepancy vector and let us limit ourselves only to the first terms:

$$\varepsilon_i = \varepsilon_i(t) = \varepsilon_i(t^0) + \sum_j^N \partial\varepsilon_i/\partial t_j\,\Delta t_j \qquad (9)$$

or in vector form:

$$\varepsilon(t) = \varepsilon(t^0) + J\Delta t, \qquad (10)$$

TABLE 1. Influence Factors for Various Parameters, Applicable to the Temperature i-th Region

Paramater	Notation	Expression of influence coefficient
Conductivity between j-th and ℓ-th bodies	$\sigma_{j\ell}$	$(b_{ij} - b_{i\ell})(t_\ell - t_j)$
Conductivity from j-th body to the medium	σ_{jm}	$b_{ij}(t_m - t_j)$
Heat flow	P_j	b_{ij}
Heat capacity	C_j	$[(dt_j)/d\tau]b_{ij}$
Temperature of the medium	t_m	$\sum_j^N b_{ij}\sigma_{jm}$

where the Jacobi matrix J has the form:

$$J = \begin{vmatrix} \partial\varepsilon_1/\partial t_1, & \ldots, & \partial\varepsilon_1/\partial t_N \\ \cdot & \cdot\ \cdot\ \cdot\ \cdot & \cdot \\ \partial\varepsilon_N/\partial t_1, & \ldots, & \partial\varepsilon_N/\partial t_N \end{vmatrix}. \qquad (11)$$

For linear systems the Jacobi matrix coincides with the matrix of thermal conductivities.

Since t^0 represents the solution of system (1) or (2) when $\Delta x = 0$, then $\varepsilon(t^0) = 0$. In this case

$$\Delta t = J^{-1}\varepsilon(t), \qquad (12)$$

where J^{-1} is the inverse Jacobi matrix.

With consideration of (12), the expression for the temperature assumes the form

$$t(x_0 + \Delta x) = t^0 + \Delta t = t^0 + J^{-1}\varepsilon(t) \qquad (13)$$

or for the i-th equation

$$t_i = t_i^0 + \sum_k^N b_{ik}\varepsilon_k(t), \qquad (14)$$

where b_{ik} is an element of the inverse Jacobi matrix.

Differentiating (14) with respect to x_j, we find

$$\frac{\partial t_i}{\partial x_j} = \sum_k^N b_{ik}\frac{\partial\varepsilon_k}{\partial x_j} + \sum_k^N \frac{\partial b_{ik}}{\partial x_j}\varepsilon_k. \qquad (15)$$

Assuming the Jacobi matrix to be independent of the parameter x_j (i.e., $\partial b_{ik}/\partial x_j = 0$), from (15) we obtain the expression for the influence coefficients

$$k_{ij} = \frac{\partial t_i}{\partial x_j} = \sum_k^N b_{ik}\frac{\partial\varepsilon_k}{\partial x_j}. \qquad (16)$$

Let us examine the determination of the coefficients of influence on the example of thermal conductivity. The derivatives of the components ε_j and ε_ℓ with respect to conductivity $\sigma_{j\ell}$ in accordance with (7) or (8) are equal to:

$$\frac{\partial\varepsilon_j}{\partial\sigma_{jl}} = \frac{\partial[\sigma_{jl}(t_l - t_j)]}{\partial\sigma_{jl}} = t_l - t_j, \qquad (17)$$

$$\frac{\partial\varepsilon_l}{\partial\sigma_{jl}} = \frac{\partial[\sigma_{jl}(t_j - t_l)]}{\partial\sigma_{jl}} = t_j - t_l.$$

Let us substitute (17) into (16):

$$k_i(\sigma_{jl}) = b_{ij}(t_l - t_j) + b_{il}(t_j - t_l) = (b_{ij} - b_{il})(t_l - t_j).$$

Expressions for the coefficients of influence with respect to such other parameters as are shown in Table 1 have been derived in analogous fashion.

Using the formulas presented in Table 1 and the rule for differentiation of complex functions, we will write the coefficients of influence for other parameters. For example, for the angular coefficient φ_{jl}

$$\frac{\partial \varepsilon_k}{\partial \varphi_{jl}} = \frac{\partial \varepsilon_k}{\partial \sigma_{jl}} \frac{\partial \sigma_{jl}}{\partial \varphi_{jl}} = \frac{\partial \varepsilon_k}{\partial \sigma_{jl}} \frac{\partial [\varepsilon_{red}\, \varphi_{jl} S_i f(t_j, t_l)]}{\partial \varphi_{jl}} = (b_{ij} - b_{il})(t_l - t_j)\, \varepsilon_{red} S_i f(t_j, t_l).$$

Let us now turn to the solution of the second problem. We will examine the analysis of the influence exerted by the parameters on the distribution of temperature in individual regions.

The temperature distribution is a function of the mean-surface temperatures t_{sk} ($k = 1, \ldots, N$) through the regions of the object, as well as of the physicogeometric parameters x_j ($j = 1, \ldots, n$):

$$t_i = t_i(t_{s1}, \ldots, t_{sN}, x_1, \ldots, x_n). \tag{18}$$

Expanding (18) into a Taylor series and retaining the first terms in the expansion, we obtain

$$\Delta t_i = \sum_j^n \partial t_i/\partial x_j\, \Delta x_j + \sum_k^N \partial t_i/\partial t_{sk}\, \Delta t_{sk}. \tag{19}$$

Substitution into (19) of the expression for Δt_{sk}, derived from (6), gives us

$$\Delta t_i = \sum_j^N \frac{\partial t_i}{\partial x_j} \Delta x_j + \sum_k^N \frac{\partial t_i}{\partial t_{sk}} \sum_j^m k_{ij} \Delta x_j \tag{20}$$

or

$$\Delta t_i = \sum_j^{M=n+m} \left(\frac{\partial t_i}{\partial x_j} + k_{ij} \sum_k^N \frac{\partial t_i}{\partial t_{sk}} \right) \Delta x_j, \tag{21}$$

or

$$\Delta t_i = \sum_j^M r_{ij} \Delta x_j,$$

where

$$r_{ij} = \frac{\partial t_i}{\partial x_j} + k_{ij} \sum_k^N \frac{\partial t_i}{\partial t_{sk}}. \tag{22}$$

In (22) r_{ij} represents the influence coefficient of the j-th parameter on the temperature distribution in the i-th region.

The partial derivative $\partial t_i/\partial x_j$ and $\partial t_i/\partial t_{sk}$ are found analytically or numerically, depending on the method used to analyze the temperature distribution in the regions.

The error in the calculation of the influence coefficients and the deviations in temperation are determined by the remaining terms of Taylor series expansions (4), (9), and (19), and it is also based on the assumption that the Jacobi matrix is independent of the parameter x_j.

According to the analysis, for linear systems [$\sigma \neq \sigma(t)$] the error in the calculation of the influence coefficients and the temperature deviations is equal to zero for the following parameters: the power P of heat release, the temperature t_m of the medium, and the heat capacity C. For thermal conductivities the analysis of the error is accomplished in statistical fashion. The matrix of the conductivities and the power vector are specified in random fashion, as the product of certain nominal values of σ_0 and P_0 multiplied by a random number worked out by a pseudorandom-number selector in the interval from zero to unity in accordance with a uniform distribution law. The temperature deviations were determined numerically as well in the proposed method. In the latter case, the temperature was calculated twice: both in the presence of and in the absence of deviations in the parameters, while the deviations in temperature of $\Delta\tilde{t}_i$ for the i-th region were found as the difference between the derived temperature values. The relative error δ_i was determined from the following formula:

$$\delta_i = \mathrm{mod}\,(\Delta\tilde{t}_i - \Delta t_i)/\Delta\tilde{t}_i. \tag{23}$$

With the results from 1000 selections, we obtained reliable intervals for the relative error of the method.

The analysis showed that the indicated error does not depend on the nominal values of σ_0 and P_0 that have been chosen (this can be demonstrated analytically) and it is determined by the number of equations and by the magnitude of the parameter deviations. For linear systems of 3, 5, 7, and 10 equations, given a 10%-parameter deviation, the relative error amounted, respectively, to 9, 6.5, 5, 4%. The dependence on the magnitude of the deviations is directly proportional.

For nonlinear systems (we examined objects whose nonlinearity was determined by radiative heat exchange) the error is larger and amounts to 13 and 5%, respectively, for 3 and 10 equations.

The approach covered here was used to analyze influence in the design of a number of optical-electronic instruments* and made it possible to determine both structural and regime parameters which ensured a normal thermal regime for these instruments. Among such parameters we can cite the following: the power of the heater for the objective thermostating system, the area and mass of the radiator in the radiation-receiver cooling system, the parameters of the mounting element (brackets, fittings, etc.).

The area in which this method can be utilized is not limited to optical and opticoelectronic instruments. It can be used successfully in the design of a variety of radioelectronic equipment (multiunit racks, power-supply units, and others) and also allows us to develop certain recommendations, namely: with respect to the positioning of individual units in racks, with respect to the determination of the required rate of air flow through the units, for the selection of radiators to cool elements in the power-supply units, etc.

Experience with the utilization of this approach for purposes of analyzing the influence of the parameters on the thermal regime of variously designated objects showed that it allows us to eliminate the need to perform a multitudinous variety of calculations, thereby saving computer time and reducing the time required for the development. These coefficients of influence are determined without additional expenditure of computer time (the inverse Jacobi matrix required for this purpose was found in the calculation of the temperatures by the Newton–Rafson method [13].

NOTATION

t_i^0, Δt_i, the i-th component of the temperature vector, calculated for nominal values of the parameters and of the correction-factor vector; τ, time; ε_{red}, reduction emissivity; S_j, area of j-th radiation surface; $f(t_j, t_\ell)$, the function relating the temperatures t_j and t_ℓ in the expression for radiative conductivity.

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STUDYING THE TEMPERATURE FIELD IN THE RECORDING AND REPRODUCTION OF INFORMATION BY MEANS OF FOCUSED RADIATION

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The functions of an instantaneous spot source of heat acting on the boundary of layer separation have been constructed for a two-layer plate. The temperature field generated by a moving normally distributed source of radiation is studied in the recording and reproduction of information.

The most important component in the development of optical disk recording devices, as well as in the reproduction and storage of information is the study of the process involved in the propagation of heat generated in an active layer applied to a substrate transparent to optical radiation and focused with brief pulsed radiation (the thickness of the substrate considerable exceeds the thickness of the active layer). Under real conditions, since the three-dimensional distribution of radiation intensity is described by a complex law [1], it is a good idea to make it as simple as possible. In this connection, of practical interest is an examination of the problem pertaining to the heating of component parts in three-dimensional formulation from the standpoint of the heat sources which are effective at the point at which the layers are joined.

The solution of these problems for a two-layer plate can be found by means of the functions $G(r, r_0, \varphi, \varphi_0, z, \tau)$, satisfying the following equation, with discontinuous and singular coefficients:

$$\Delta G+\left(1-\frac{\lambda_1}{\lambda_2}\right)\frac{\partial G}{\partial z}\bigg|_{z=z_1-0}\delta(z-z_1)-\left[\frac{1}{a_1}+\left(\frac{1}{a_2}-\frac{1}{a_1}\right)S(z-z_1)\right]\frac{\partial G}{\partial \tau}+ \\ +\frac{1}{\lambda_2}\frac{\delta(r-r_0)}{r_0}\delta(\varphi-\varphi_0)\delta(z-z_1)\delta(\tau)=0 \quad (1)$$

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